FRAMEWORK OF A DYNAMIC CYCLE FOR PROMOTING THE PROFESSIONAL DEVELOPMENT OF MATHEMATICS TEACHERS AND EDUCATORS IN THE LESSON STUDY OF SCHOOL MATHEMATICS*

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Abstract

A fundamental problem in mathematics education is "What can we do for students to enhance their mathematical ability and achievement and how can we do it?" This paper searches for a promising solution to this problem. To that end, the author focuses on the lesson study approach, specifically adopting the same approach adopted by the mathematician George Polya, who reflected on his experience and described his methods of mathematical problem solving. As a mathematics educator, the author shares his experience of two types of lesson study of primary school mathematics, and proposes a framework of a dynamic cycle in the lesson study of school mathematics for promoting the professional development of mathematics teachers and educators. This professional development may improve the mathematical ability and achievement of students at schools.

Key words: mathematics education, lesson study, professional development, dynamic cycle, reflection

INTRODUCTION

The theme of the conference PME 40, held in Szeged, Hungary, in 2016, was 'Mathematics Education: How to solve it?' Relating to this theme and as a fundamental problem to be addressed and solved in mathematics education, the author poses the educational problem: "What can we do for students to enhance their mathematical ability and achievement and how can we do it?" This paper searches for a promising solution to this problem by focusing on the approach of the lesson study of school mathematics because the author considers that the quality of education cannot exceed the quality of teachers. Specifically, the author adopts the same approach adopted by the mathematician George Polya, who reflected on his mathematical experience and described his methods of mathematical problem solving (Polya, 1945).

The lesson study of school mathematics has been recognized as an important cultural and collaborative means of the continuous professional development of teachers as a life-long process in Japan (National Association for the Study of Educational Methods, 2011). The Japanese lesson study model is well known internationally (Stigler & Hiebert, 1999; Lewis, 2002; Isoda, et al., 2007; Shimizu, 2010; Takahashi, 2010, 2014). During lesson study, teachers collaborate to 1) formulate long-term goals for student learning and

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development; 2) plan and conduct lessons based on research and observation to apply these long-term goals to actual classroom practices for particular academic contents; 3) carefully observe the levels of students' learning, their engagement, and their behaviours during the lesson; and 4) hold post-lesson discussion with their collaborative groups to discuss and revise the lesson accordingly (Lewis, 2002). The success of lesson study primarily depends on improvements in teacher practice and the promotion of collaboration among teachers. Lesson study provides Japanese teachers with opportunities to make sense of educational ideas within their practice, to change their perspectives about teaching and learning, and to learn to see their practice from the students' perspective (Takahashi, 2010). The lesson study can link together and promote both the pre-service teacher training at universities and the in-service teacher training at school, local district, and nationwide levels (Koyama, 2008b; Corey et al., 2010). The lesson study as in-service teacher training takes different forms.

In the following sections, as a mathematics educator who has been actively involved in various lesson studies in Japan, the author shares his experience of two types of lesson study: a school-based and district-wide lesson study (Koyama, 2010b) and a school-based and cross-district lesson study (Koyama, 2015) of problem-solving lesson as part of primary school mathematics. The author then reflects on the whole process of case studies of lesson study by focusing on the relationship between collaboration and reflection. Finally, as a promising solution to the problem posed in this paper, the author proposes a framework of a dynamic cycle for promoting the professional development of mathematics teachers and educators in the lesson study of school mathematics that may improve students' mathematical ability and performance.

BACKGROUND

Before sharing the author's experience of lesson study, we look at the education system in Japan and the problem-solving lesson model of the teaching and learning of school mathematics (Koyama, 2008a, 2010a) to clarify and reflect on the lesson study of problem-solving lesson as part of primary school mathematics in Japan.

Education system in Japan

The school education system in Japan comprises six years of education at primary school, three years at lower secondary school, and three years at upper secondary school. The nine years spent at primary and lower secondary schools, for students aged from 6 years to 15 years, are compulsory education for all children. The national curriculum standard is prescribed in the Course of Study determined and issued by the Ministry of Education (e.g., Ministry of Education, 2008a, 2008b, 2009). School textbooks must be approved by the Ministry of Education in line with the Course of Study. Public school teachers are local prefectural or municipal officials and are appointed by the respective local prefectural or municipal boards of education. Primary school teachers teach almost all school subjects at their own grade while secondary school teachers teach their major school subjects.

Problem-solving lesson as a typical model of good practice in Japan

A typical model of good practice recognized by many Japanese educators and teachers is the problem-solving lesson in the mathematics classroom (Becker & Shimada, 1997; Stigler & Hiebert, 1999; Burghes & Robinson, 2009; Koyama, 2012). The problem-solving lesson has four distinct phases: presentation of the problem, development of a solution, progression through discussion, and summarizing the lesson (Burghes & Robinson, 2009, p.56). This might be characterized as the parallel and collaborative version of four steps identified by Polya (1945) in solving a mathematical problem: understanding the problem, devising a plan, carrying out the plan, and looking back and extending.

Why do many Japanese educators and teachers recognize the problem-solving lesson as a good model for the teaching and learning of mathematics? Sawada (1997) pointed out the advantage of using the problem-solving lesson model for teaching and learning mathematics in the classroom as follows.

> Students participate more actively in the lesson and express their different ideas or solutions more frequently. Students have more opportunities to make comprehensive use of their knowledge, skills, and ways of thinking. Even low achieving students can respond to the problem in some significant ways of their own. Students are intrinsically motivated to give their justifications or proofs. Students have rich experience in the pleasure of mathematical activities and receive the approval from peer students in the classroom (Sawada, 1997, p.23).

The essence of doing mathematics is the process of solving a problem mathematically rather than a product. If acquired in the process of solving problems mathematically, then we believe that the mathematical knowledge, skills, and ways of mathematical thinking are applicable in a new or unfamiliar situation for learners. In the mathematics classroom, therefore, a mathematics lesson as an integration of the teacher's teaching activity and the students' learning activity should be structured for the overall process of solving problems mathematically. The problem-solving lesson is a student-centred model of teaching and learning mathematics that may encourage students to construct mathematics collaboratively in a mathematics classroom using their naïve conceptions as well as their acquired mathematical knowledge, skills, and ways of mathematical thinking.

In the following three sections, the author shares his experience of two types of lesson study at primary schools at which he had been involved as an external or internal expert of mathematics education: a school-based and district-wide lesson study (Koyama, 2010b) and a school-based and cross-district lesson study (Koyama, 2015). These two types of lesson study of school mathematics are then compared by taking examples of research lessons from case studies of primary school mathematics.

CASE STUDY 1
SCHOOL-BASED AND DISTRICT-WIDE LESSON STUDY

In this section, as the first case study, we look at the school-based and district-wide lesson study of primary school mathematics at an Onomichi City Public Primary School (OCPPS) where the author, as an external expert of mathematics education, has collaboratively worked with school teachers for three years

(Koyama, 2010b). This case study is a typical example of the school-based lesson study in local public schools, while the lesson study at the OCPPS is not only school-based but also district-wide. The mathematics lessons at the OCPPS were structured using the problem-solving lesson model for the teaching and learning of primary school mathematics to foster students' ability to think and represent mathematically.

Characteristics of the OCPPS

The OCPPS is located in Onimichi City in Hiroshima Prefecture. The school had 260 students and 14 teaching staff: 10 classroom teachers including two teachers for students with special needs, one chief of the instruction department, one chief of the research department, one school vice-principal, and one school principal. The school was designated by the Onomichi City Board of Education as a pilot school for primary school mathematics in the city for three academic years from April 2008 to March 2011. The pilot school for mathematics was expected to conduct practical research on mathematics education through lesson study for improvement of both the students' mathematical achievement and the teachers' classroom practice, and to share the results and experience with many teachers from other primary schools in the city district.

Overall process of the lesson study at the OCPPS

The overall process of the lesson study for each academic year at the OCPPS consisted of identifying a practical research theme for the lesson study, planning the lesson study for a year, forming two groups of teachers for the lesson study, conducting cycles of the lesson study, and opening classes to teachers from other schools in the city district as follows.

Identifying a practical research theme for the lesson study

In April 2008, a committee organized by the principal, the vice-principal, and the chief of the research department identified the practical research theme of the lesson study for three years: fostering students' ability to think and represent mathematically. The theme reflected the emphasis of the Course of Study at that time (Ministry of Education, 2008a). It also considered students' issues in learning mathematics by analysing the results of mathematics achievement tests.

Under the three-year theme, for the first year, the committee decided to explore how to improve mathematics lessons for students in terms of enhancing the students' mathematical thinking. In particular, the committee focused on students' mathematical activity, devising mathematics notebooks, and collaborative learning to accomplish the first-year goals.

Planning the lesson study for a year

The school adopted the lesson study cycle of: 1) investigating a variety of teaching and learning materials; 2) developing a lesson plan; 3) conducting a research lesson; 4) holding a post-lesson discussion; and 5) revising the lesson plan for the improved classroom practice (Koyama, 2010b). By incorporating the lesson study cycles into one academic school year calendar, the school made the schedule of the lesson study for one year. At the beginning of the academic year, there were several sessions for all teachers to share the research theme and goals for one year at the school, to understand students' actual performance and issues in learning primary school mathematics by analysing the results of national and school tests, and to come to an

agreement on how to regularly conduct research lessons for one year at the school.

Forming two groups of teachers for the lesson study

The school had 12 teachers with teaching experience varying from just 2 years to over 30 years. The teachers were thus divided into two groups with due regard to their teaching experience and their students' grade. In the third year of the project, one group of teachers was expected to conduct lesson studies on topics in the content areas of "Numbers and Calculations" and "Geometrical Figures" while the other group was expected to conduct lesson studies on topics in the content areas of "Quantities and Measurements" and "Mathematical Relations" even though each teacher at a primary school teaches all mathematical topics in the four content areas using primary school mathematics textbooks and teachers' resource books and tools.

Conducting cycles of the lesson study

Before conducting a research lesson, the lesson planning team developed a lesson plan for a topic. In the process of developing the lesson plan, a teacher who would conduct a research lesson first wrote a draft of the lesson plan. Then, in a session on developing the lesson plan, all members of the same team examined the draft from their own viewpoints and exchanged their ideas and experience among the team members to improve the draft of the lesson plan.

During the research lesson conducted by a classroom teacher of the team, all participants observed the research lesson and collected data of the lesson to evaluate and improve the classroom practice and lesson plan. The school created a special sheet for participants to check the observed lesson in terms of objectives of the lesson and to take notes on both the teacher's classroom practice and the levels of students' learning and the students' behaviours during the lesson. The author as an expert of mathematics education was regularly invited to participate in the lesson studies at the school.

In the post-lesson discussion session, copies of the record of the research lesson were distributed to all participants. One teacher acted as the chair of the session and another teacher as the note taker of the session. After a brief explanation of the lesson plan and the conductor's short comment on his/her teaching practice, participants were divided into two groups to discuss the lesson in terms of good practice, issues to be addressed, and the proposal of possible ideas and strategies for improving the classroom practice and the lesson plan using the data recorded on their evaluation sheet and the records of the lesson. After the group discussion, the leader of each group reported to all participants the essence of the group discussion with a poster. At the end of the discussion session, the author as an external expert of mathematics education was given an opportunity to make a summarizing comment on the lesson study.

Opening classes to teachers from other schools in the city district

The OCPPS as a pilot school was required once a year to open all classes to teachers from other schools in the city district. After several cycles of lesson study, all teachers developed their lesson plan for the open classes. During the summer vacation for students, each teacher drafted a lesson plan, and the group of teachers then examined and discussed their drafts with each other to improve the drafts. The author was asked to check all the drafts for open classes and visited the school to hold discussions with the teachers to improve their drafts of the lesson plan. In the process of developing lesson plans for the open classes,

teachers had opportunities to reflect on their lesson studies conducted in the previous semester and to make use of their experience and 'lessons' learnt through conducting lesson study at the school. Meanwhile, teachers from other schools in the city district had opportunities to observe lessons and participate in the post-lesson discussion sessions to share different ideas for improving mathematics instruction in their classroom. At the end of the school year, the school made a final report by summarizing the results and tasks not only for reflection on the one-year lesson study but also for preparation of the next year (Koyama, 2010b).

CASE STUDY 2
SCHOOL-BASED AND CROSS-DISTRICT LESSON STUDY

As the second case study, this section looks at a school-based and cross-district lesson study of primary school mathematics at the Hiroshima University Attached Primary School (HUAPS), where the author as a colleague and an internal expert of mathematics education has been collaboratively working with all the mathematics teachers of the school for several decades since the 1990s (Koyama, 2015). This case study is a unique example of the school-based and cross-district lesson study in notional primary schools, because the lesson study in the department of primary school mathematics at the HUAPS involves not only mathematics teachers and mathematics educators but also graduate school students attending PhD and Master's courses on mathematics education at Hiroshima University. The mathematics lessons at the HUAPS have been structured using the problem-solving lesson model for the teaching and learning of primary school mathematics in emphasizing students' continuous awareness of learning mathematics.

Characteristics of the HUAPS

The HUAPS is located in Hiroshima City in Hiroshima Prefecture. The school has been attached to Hiroshima University for more than 100 years and is tasked with taking part in practice teaching for pre-service teacher training at Hiroshima University, performing new developmental research for next-generation education, and conducting lesson study and opening classes to teachers from all districts in Japan. The school is a unique national primary school because it has three mathematics teachers who teach only mathematics at different grades, while other primary school teachers teach almost all school subjects at their own grade at national schools. For more than 100 years, the school has issued its own monthly journal *School Education*. Each edition includes a pair of articles on the lesson study of primary school subjects written by one HUAPS teacher who conducted a research lesson and one Hiroshima University educator who is a specialist in school subject education. In addition, for a periodic seminar on primary school mathematics, we have organized a collaborative study group *Primary School Mathematics Seminar* comprising not only mathematics teachers and mathematics educators but also graduate school students attending PhD and Master's courses on mathematics education at Hiroshima University.

Overall process of the lesson study at the HUAPS

We present one example of lesson studies conducted by the collaborative study group *Primary School*

Mathematics Seminar at the HUAPS from November 2012 to February 2013 in the third semester of the 2012 academic year (Koyama, 2015). The lesson adopted the problem-solving lesson model in the teaching and learning of the topic "Triangle" for second graders at the school. The overall process of the lesson study at the HUAPS comprised seminars before the research lesson, planning the research lesson, conducting the research lesson and post-lesson discussion, and reflection on the research lesson as follows.

Seminars before the research lesson

In November 2012, one collaborative study team of *Primary School Mathematics Seminar* began a collaborative study on teaching materials to be used in the teaching and learning of "Triangle" in the teaching unit "Triangle and Quadrangle" for second graders at primary school. There were four 90-minute sessions from November 2012 to January 2013 before the research lesson. The team was organized by the mathematics teacher who would conduct the research lesson, three graduate school students of mathematics education attending PhD and Master's courses, and two mathematics educators at Hiroshima University. The team had conducted collaborative studies on the teaching materials to help the mathematics teacher design the teaching unit "Triangle and Quadrangle" and develop the research lesson plan of "Triangle" for his second graders.

Planning the research lesson

After the four sessions, Mr. Maeda, the mathematics teacher of the research lesson, designed a series of 15 lessons in the teaching unit "Triangle and Quadrangle" for his second graders. He then selected the fifth of the 15 lessons as the research lesson and developed the lesson plan of the research lesson.

Conducting the research lesson and post-lesson discussion

The research lesson was conducted by Mr. Maeda with his 40 classroom students on 9th February 2013 as one of the open classes at the annual Open School Conference. Approximately 400 teachers from many districts attended the conference and about half of them observed this lesson. Immediately after the research lesson, a post-lesson discussion was held with the participants, where the teacher explained briefly the objectives of the research lesson and gave his short reflection on the lesson and there was then 30 minutes of questions and answers among participants. The research lesson and post-lesson discussion were recorded on video and photographed by the graduate school students of the collaborative team, and the author observed the lesson and took field notes.

Reflecting on the research lesson and writing articles on the lesson study

After the Open School Conference, the detailed transcript of the research lesson was made available for both the teacher and the author to reflect on the research lesson and write a pair of articles of the lesson study that were submitted to the monthly journal *School Education*, issued by the HUAPS. This final process of writing a pair of articles is very unique to Case Study 2 at the HUAPS. About 1 year later, the pair of articles was published in the January 2014 issue (Koyama, 2014; Maeda, 2014). The publication of the pair of articles on the teacher's (Maeda, 2014) and educator's (Koyama, 2014) reflections on the lesson study is important in the sense that the articles promote the lesson study of school mathematics not only as a means of the continuous professional development of teachers but also as an authentic research area in the science

of mathematics education.

COMPARISON OF TWO TYPES OF LESSON STUDY

Table 1 compares two types of lesson study using an example from each of the two case studies on primary school mathematics.

The research lesson in Case Study 1 was conducted by a female teacher with a few years of teaching experience. Before conducting the lesson, the lesson planning team of school teachers developed the lesson plan of "Multiplication in Vertical Form" for the third grade. The objectives of the lesson were to develop the students' understanding that multiplication can be calculated using the multiplication rule: the product of multiplication does not change if we multiply by decomposing a multiplicand or a multiplier into some numbers. The features of the lesson were posing the problem of how to calculate 213×3 by applying a previously learnt method of multiplication in vertical form and focusing on numerical positions, using Japanese coins to calculate multiplication on the basis of the place value system of decimal notation, and conducting two types of exercises at the end of the lesson; e.g., an exercise in which a three-digit number is multiplied by a one-digit number and an exercise in which students work in pairs to solve a missing-value problem and explain their solution strategy. From the external expert observer's perspective, it can be said that the teacher had improved her classroom practice through the collaborative lesson studies with her colleagues at the school in one year.

The research lesson in Case Study 2 was conducted by a male teacher with 18 years of teaching experience and a Master's degree in education. After the four seminar sessions held by the collaborative study team, he designed a series of 15 lessons for the teaching unit "Triangle and Quadrangle" for his second graders of the school. He then selected the fifth of the 15 lessons as the research lesson and developed a lesson plan for the research lesson. The objectives of the lesson were to foster students' ability to think logically, especially through activity that stimulates the students to think about the definition of a triangle. The features of the lesson were to use different figures as teaching materials, to show those figures one by one and thus stimulate students to remember the definition of a triangle and to use explicitly the definition in explaining the reason for their judgment, and to incorporate the important question "Why would a figure that has three connected points not be a triangle?" to shake the students' recognition of a triangle and deepen the students' understanding of the definition of a triangle. From the internal expert observer's perspective, it can be said that the teacher had improved his classroom practice in terms of developing the teaching materials through the collaborative lesson studies with his study team members of *Primary School Mathematics Seminar* at the school in one year.

REFLECTION ON THE LESSON STUDY IN CASE STUDY 2

In the following, we reflect on the details of the lesson study in Case Study 2 from three perspectives: the collaborative study of teaching materials for the research lesson, the development of a lesson plan for the

Table 1: Comparison of two types of lesson study with examples taken from research lessons

Characteristics	Case Study 1: OCPPS	Case Study 2: HUAPS
Type of Lesson Study	School-based and district-wide	School-based and cross-district
Overall Process of Lesson Study	1. Identifying a practical research theme for the lesson study 2. Planning the lesson study for a year 3. Forming two groups of teachers for the lesson study 4. Conducting cycles of the lesson study 5. Opening classes to teachers from other schools in the city district	1. Seminars before the research lesson 2. Planning the research lesson 3. Conducting the research lesson 4. Post-lesson discussion on the research lesson 5. Reflecting on the lesson study and writing articles on the lesson study
Teacher of Research Lesson	A female teacher with a few years of teaching experience	A male teacher with 18 years of teaching experience
Author's Position in Lesson Study	An external expert of mathematics education in giving suggestions for a lesson plan, observing a research lesson, and making a comment on the lesson in the post-lesson discussion	An internal expert of mathematics education in making a lesson plan, observing a research lesson, and writing an article of the reflection on the lesson study issued in a journal
Teaching Unit and Grade	· "Multiplication in Vertical Form" in the third grade · 37 students	· "Triangle and Quadrangle" in the second grade · 40 students
Objectives of Research Lesson	· To develop students' understanding that multiplication can be calculated using the multiplication rule: the product of multiplication does not change if we multiply by decomposing a multiplicand or a multiplier into some numbers	· To foster students' ability to think logically especially through activity that stimulates the students to think about the definition and properties of geometric figures · To deepen students' recognition and understanding of a triangle
Features of Research Lesson	· Using Japanese coins to calculate multiplication on the basis of the place value system of decimal notation · Conducting students' activities of explaining with pictures and calculating separately for each position of the multiplicand to understand and explain the meaning of multiplication	· Using different figures as teaching materials and showing these figures one by one and thus stimulate students to remember the definition of a triangle and to use explicitly the definition in explaining the reason of their judgment · Incorporating the important questioning "Why would a figure that has three connected points not be a triangle?" to shake students' recognition of a triangle and deepen students' understanding of the definition of a triangle
Perspectives for the Analysis of and Reflection on Lesson Study	· How to calculate a multiplication such as 213×3 by applying a previously learnt method of multiplication in vertical form and focusing on numerical positions · Two types of exercises at the end of the lesson; an exercise in which three-digit number is multiplied by a one-digit number and an exercise in which students work in pairs to solve a missing-value problem and explain their solution strategy	· Viewpoints of a figure: To classify and identify the given figures from different viewpoints of the figure · Using counterexamples: To understand the definition of a triangle through the discussion with counterexamples · Different roles of teacher: To play two different role of 'supportive' and 'active' interventions to promote social interactions among the students

research lesson, and the reflective analysis of the research lesson. The reason why we reflect on the lesson study in Case Study 2 is the lesson study involved graduate school students who would become mathematics teachers or mathematics educators. The lesson study can link together and promote both the pre-service teacher training at universities and the in-service teacher training at schools, which may improve the mathematical abilities and achievement of students.

Collaborative study of teaching materials for the research lesson

In the first session of the seminar, the mathematics teacher proposed the students' activity of identifying whether a figure is a triangle so that his students capture the concept of a triangle in terms of its components and shared with the team members his expectations to be realized through this activity. The main issue of the team discussion was what criteria should students use in judging whether a figure is a triangle. Finally, they recognized the necessity to investigate the mathematical background related to the definition of a triangle and the research findings on the merits and demerits of using counterexamples for the formation of the students' concept of a triangle.

In the second session, as a result of the team discussion, it was confirmed that the activity proposed by the mathematics teacher in the previous session could help the students learn a fundamental mathematical activity of going back to the definition if needed. Through the activity, the students might be encouraged to think about the reason why a triangle is not defined as a figure with three connected points. The team members then discussed whether an alternative teacher's activity of asking the students "Can you see a triangle in the given figure?" would be more appropriate for the students than the asking them "Is the given figure a triangle?" At the end, they recognized the necessity to design the development of a research lesson in more detail.

In the third session, it was confirmed that the planned research lesson could be theoretically supported by both the philosophy of mathematics education and the cognition theory of geometric figures in psychological research. The team members agreed that the research lesson could contribute to deepening the students' recognition and understanding of a triangle. They then discussed how the students would interpret the teacher's directive "Let's classify the given figures into two groups." Finally, the mathematics teacher proposed six different figures to be used in the research lesson (see Figure 1). The team members discussed an effective way of showing the figures to the students in the research lesson.

In the fourth session, the main issue of the team discussion was how to deepen students' understanding of the definition of a triangle such that a geometric figure surrounded by three straight lines is named a triangle which was already learnt in the previous lesson in the second grade. In the discussion, the team focused on how to organize both the teacher's activity of shaking students' recognition and understanding of a triangle and the students' activity of rethinking the definition of a triangle. To promote these activities, they discussed a possible way of showing the prepared six figures and the expected merits and demerits of giving the students counterexamples of a triangle in the research lesson.

Development of a lesson plan for the research lesson

After the four sessions, the mathematics teacher of the research lesson developed the lesson plan. We must remember that before this research lesson, the second graders had already learnt the definition of a

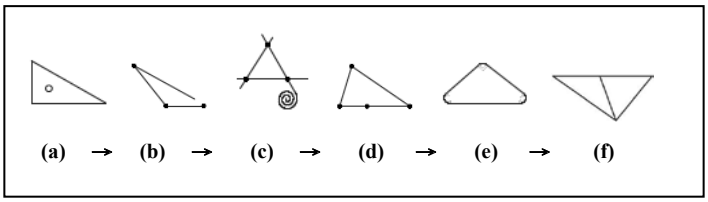

Figure 1: Intended order of showing the figures to the students in the research lesson

triangle in the teaching unit "Triangle and Quadrangle" such that a geometric figure surrounded by three straight lines is named a triangle. There are three remarkable features in the lesson plan. The first feature is the use of the different figures shown in Figure 1, including the intricate figures of Fig. 1(c) and Fig. 1(d), as teaching materials. The second is the way the figures are shown to the students one by one in the order of Fig. 1(a), Fig. 1(b), Fig. 1(c), and so on (see Figure 1) to encourage the students to remember the definition of a triangle and to use the definition in explaining explicitly the reason for their judgment. The third is the incorporated important question "Why would a figure with three connected points not be a triangle?" to shake the students' recognition of a triangle and to deepen their understanding of the definition of a triangle. It might be said that the three features are crystallized as a result of the collaborative team study on teaching materials before conducting the research lesson.

Reflective analysis of the research lesson

The research lesson was analysed by both the teacher and the author as an internal expert of mathematics education using the detailed transcript and a photograph of the blackboard used in the lesson. In the reflective analysis of the research lesson, referring to the teacher's reflection on the research lesson the author identified the importance of emphasizing viewpoints of the figure and using counterexamples and the different roles played by the teacher in two notable scenes of the lesson (Koyama, 2014, 2015). In the first notable scene, where Fig. 1(c) was shown by the teacher, the students had opposing judgments about whether the figure is a triangle. After students talked for 1 minute in pairs, they actively exchanged their viewpoints of Fig. 1(c) in a whole-classroom discussion. Some students presented their opinions in their own words. For example, one student said "Those who judge Fig. 1(c) as not a triangle see the figure as a whole, including the spiral part. Meanwhile, those who judge Fig. 1(c) as a triangle see the inside part of the figure surrounded by three straight lines." During the classroom discussion, the teacher did not make any comments about the students' opinions and only wrote down keywords related to their viewpoints of the figure on the blackboard; e.g., "If we change our viewpoints", "It depends on the viewpoint of the figure", and "If we see the figure like this" (see Figure 2). In this scene, the teacher's supportive intervention functioned effectively for the students to exchange their judgments by referring to the definition of a triangle and to share different viewpoints of the same Fig. 1(c).

The second notable scene in the research lesson relates to the discussion between the teacher and students. The scene began with the teacher's questioning of the students "Why would a figure with three connected points not be a triangle?" Immediately many students objected to what the teacher said. A student explained why a figure with three connected points might not be a triangle by drawing a figure (counterexample A) (see Figure 3) and saying, "If three points are connected by curved lines, the figure is not a triangle. Three

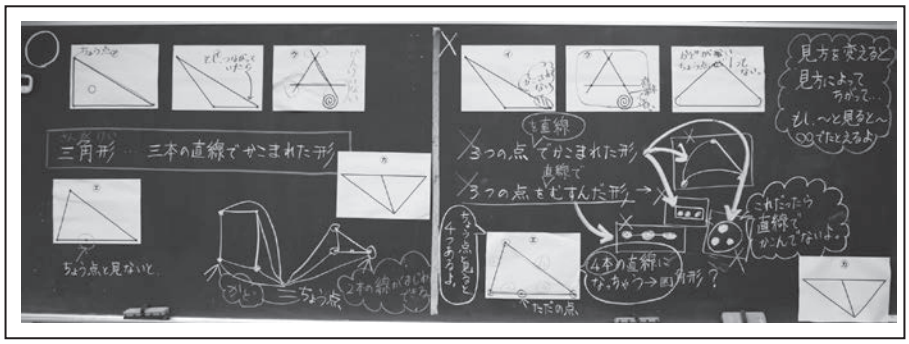

Figure 2: Photograph of the blackboard in the research lesson

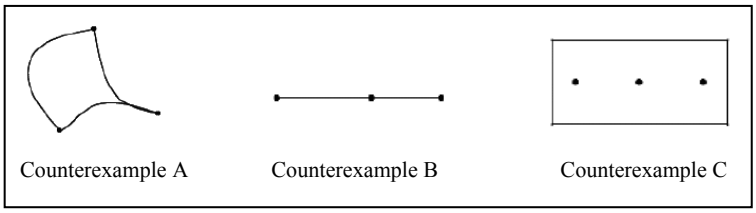

Figure 3: Counterexamples presented in the research lesson

points must be connected by straight lines." Immediately the teacher counterattacked again by saying to the students, "I see! Do you agree that a figure with three points connected by straight lines is a triangle?" Another student did not agree with the teacher and refuted what the teacher said by drawing a figure (counterexample B) with three points on the same straight line and saying, "I do not agree. If three points connected by a straight line is a triangle, then this (counterexample B) should be a triangle. Therefore, the triangle must be a figure encircling three points by straight lines." The student insisted on the importance of the figure being encircled by the straight lines. At that moment, many students seemed satisfied with their peer's refutation. However, the teacher with a smile drew another figure (counterexample C) on the blackboard. In this scene, it can be said that the teacher's active intervention functioned effectively for the students to deepen their understanding of the definition of a triangle through a whole-classroom discussion with counterexamples.

FRAMEWORK OF A DYNAMIC CYCLE

As a result of the comparison of the two case studies and the reflection on the two types of lesson study, especially the lesson study in Case Study 2 involved with not only mathematics teacher and educators but also graduate school students by focusing on the relationship between collaboration and reflection, the author insists that two types of complementary reflection function dialectically in the overall process of lesson study in both cases. The author calls these *collaborative reflection* and *individual reflection*. *Collaborative reflection* functions when a group of people reflect collaboratively on what the group did. Meanwhile, *individual reflection* functions when each person reflects individually on what the person did.

In the lesson study, for example, *collaborative reflection* functioned when all members of the lesson planning team examined the draft from their own viewpoints and exchanged their ideas and experience among the members to improve the draft of the lesson plan. In the post-lesson discussion, *collaborative reflection* functioned when the group of teachers discussed the observed lesson in terms of good practice, the issues to be improved, and the proposal of alternative ideas and strategies for improving their classroom practice and lesson plan. Meanwhile, in the lesson study, *individual reflection* functioned when, after the team discussion, the individual teacher elaborated his/her lesson plan for the research lesson, and when the teacher wrote an article on the lesson study using the detailed transcript and the photograph of the blackboard used in the lesson. The dialectic cycle of complementary *collaborative reflection* and *individual reflection* in the lesson study of school mathematics can function as a driving force for promoting the professional development of a group and an individual.

Finally, as a promising solution to the problem posed in this paper, the author proposes the framework of the dynamic cycle for the professional development of mathematics teachers and educators in the lesson study of school mathematics (see Figure 4). The dynamic cycle is driven by the dialectic cycle of two complementary reflections in lesson study for the professional development of teachers, which may improve

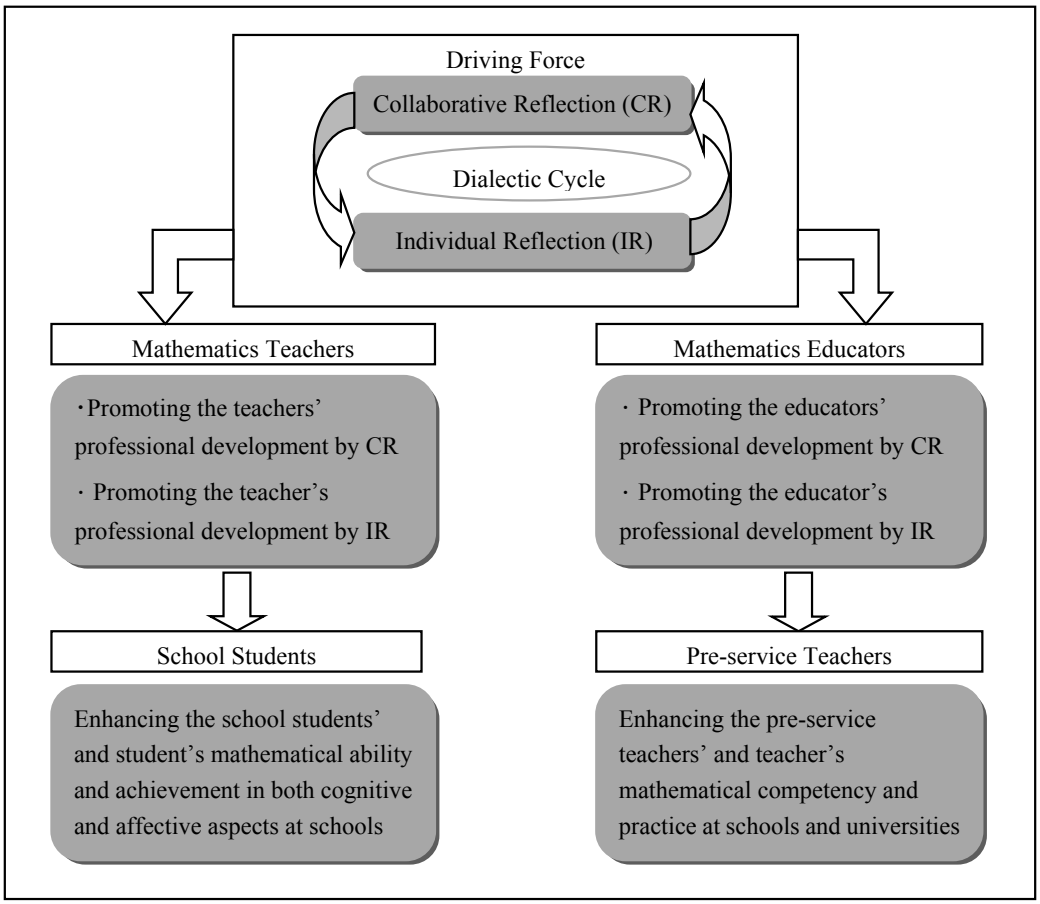

Figure 4: Framework of the dynamic cycle for the professional development of mathematics teachers and educators in the lesson study of school mathematics

both cognitive and affective aspects of the mathematical abilities and achievement of students at schools. This aspect is represented on the left of Figure 4, with the two complementary reflections put as the driving force in lesson study. Meanwhile, the right side of Figure 4 implies that the dialectic cycle promotes the professional development of mathematics educators and greatly improves the mathematical competency and teaching practice of pre-service teachers involved in the lesson study at schools and universities.

CONCLUSION

The author shared his experience of two types of lesson study for problem-solving lessons of primary school mathematics in Japan. By comparing two case studies and reflecting on the lesson study in Case Study 2 while focusing on the relationship between collaboration and reflection, the author insisted that complementary *collaborative reflection* and *individual reflection* function dialectically in the overall process of lesson study in both cases. The author characterized the dialectic cycle of the two complementary reflections as a driving force. As a promising solution to the problem posed in this paper, the author proposed a framework of a dynamic cycle driven by the dialectic cycle of two complementary reflections in the lesson study of school mathematics for the professional development of mathematics teachers, which may improve the mathematical ability and achievement of students at schools. It was also suggested that the dialectic cycle promotes the professional development of mathematics educators and greatly improves the mathematical competency and teaching practice of pre-service teachers at schools and universities.

The author insists that lesson study is not only an important means for the continuous professional development of teachers but also an authentic research area in the science of mathematics education. Therefore, in further research, it is a critical issue for all of us in mathematics education to certify how the dynamic cycle driven by the dialectic cycle of two complementary reflections in the lesson study of school mathematics can work effectively and productively in promoting the professional development of mathematics teachers and educators. We need further research on qualifying two paths represented in the both sides of Figure 4 in order to improve the mathematical abilities and achievement of students at schools and the mathematical competency and teaching practice of pre-service teachers involved in the lesson study at schools and universities.

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